HILL'S EQUATIONS AND LIOUVILLE'S FORMULA

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Periodicity (and its consequence, i.e. oscillatority in case of combined effect of more forces) is surely the basic and the most important phenomenon of nature. The differential equation has periodic solutions only if coefficients are periodic with the same period (or specially, constants). This is the necessary condition for the periodicity of the differential equation. However, the sufficient conditions are related to the properties of classes of equations and can be rather different. In this work we have offered four theorems on periodic solutions of Hill's equation, which we have not found in familiar monographic works [1] and [2]. We have shown that theorems 1^o – 4^ocan serve for establishing different classes of Hill's equations, but periodic solutions can be obtained only for very narrow sub-classes of equations and only for certain values of given coefficients in supposed particular integrals.

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1. Introduction

The equation of the following form:

$$y'' + b(x) \cdot y = 0 \tag{1.1.}$$

where *b(x)* is a periodic function, is called Hill's equation. This equation is a well-known differential equation which frequently occurs in physical, technical and astronomic problems. A great number of important equations, frequently or directly or after performing adequate transformations, belongs to the Hill's equation type. For example, these are some Legendre's equations, some hypergeometric equations and Bessel and Matthew's equations.

One of the basic questions related to the equation (1.1) is if it has periodic solutions, either one class of periodic solutions or all periodic solutions, when we say that the solutions are in co-existence.

The necessary condition for periodicity of solutions of the differential equations requires all coefficients to be periodic with the same or commensurable period. Since the coefficient *b(x)* is periodic, it means that the necessary condition is fulfilled. However, the literature does not emphasize enough the problem of the second condition, i.e. if the integral $\int b(x) \cdot dx$ is periodic or non-periodic and when this is condition for one or both solutions to be periodic.

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